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On the Nonuniqueness of the Entry Length

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The problem of entry length in laminar channel and pipe flows has a very large literature (*1*) which appears to be exclusively preoccupied with flow development starting from an initially flat or uniform velocity profile. The entry length is usually defined as the length at which the centerline velocity reaches 98% of the final value. Boundary-layer equations are generally employed, but there are exceptions (*2*). Considerable precision in this standard calculation is evident; values for channels of $(0.034 \pm 0.001)\ WN_{Re}$ are now accepted. Here W is the channel width and the Reynolds number is based on W and the mean velocity.

During some numerical experiments (*3*) we tried some nonuniform initial profiles for a channel. The program utilized simple finite-difference methods (*4*) to solve the full two-dimensional steady state form of the Navier-Stokes equations. The initial profiles tried were: (1) uniform profile; (2) symmetrical triangular profile; (3) parabolic velocity over central half of channel, zero velocity on outer parts (symmetrical profile); and (4) parabolic velocity over top half of channel, zero velocity on bottom half (asymmetrical profile).

Reynolds numbers up to 100 were tried but no calculations for $N_{Re} \gtrsim 50$ were successful. For profile 1, entry lengths of $0.4W$, $0.6W$, and $1.7W$ were recorded at Reynolds numbers of 0.1, 10, and 50, respectively. For $N_{Re} = 50$, the entry length is close to that given by the $0.034\ WN_{Re}$ formula. As $N_{Re} \to 0$, the entry profile can be shown to die away exponentially (*5*), giving an entry length of about $0.4W$; this agrees with our value for $N_{Re} = 0.1$. Profiles 1 to 4 were tested at $N_{Re} = 10$ (limited by running time), giving entry lengths of $0.6W$, $0.6W$, $0.9W$, and $1.4W$, respectively. In the latter case, the 2% error criterion is unsuitable and the length taken for the centerline axial velocity to become greater than all others was taken as being the entry length. The ironing out of the asymmetry is clearly very slow. Confirmation of these values is available from the work of Kawaguti (*6*), who has solved numerically the problem of a suddenly expanding asymmetrical channel; thus his entry profiles are natural entry profiles and are quite similar to our case (4). Kawaguti (*6*) obtained entry lengths $> 1.2W$, $> 3.2W$ for Reynolds numbers of 16 and 64, respectively. His work also ceases at an upper limit of $N_{Re} = 64$.

The conclusions are:

1. Boundary-layer theory gives reasonable results for entry length at $N_{Re} > 50$ for flat profiles.
2. The inlet length is not a unique quantity but depends upon the inlet profile.
3. Asymmetrical profiles take a very much longer time to settle down than symmetrical profiles.
4. Further research, if any, on inlet lengths should include nonuniform entry profiles. Although some of these points are perhaps obvious, we have not seen them explicitly mentioned before; flow in pipes can be expected to show similar phenomena.

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